Path following for mobile robots
Guiding Vector Fields (GVF)

Given a desired path, how does a robot follow it?
Dubin’s path consists of a sequence of lines and circles.
Implicit equations of the path

Circle $\rightarrow f(x,y) := (x-x_c)^2 + (y-y_c)^2 - r^2 = 0$

Straight line $\rightarrow f(x,y) := ax + by + c = 0$
Level sets

\[ x^2 + y^2 - r^2 = 5 \quad \text{Outer} \]
\[ x^2 + y^2 - r^2 = 0 \quad \text{Target trajectory} \]
\[ x^2 + y^2 - r^2 = -5 \quad \text{Inner} \]

\[ f(x,y) = 5 \]
\[ f(x,y) = 0 \]

\[ f(x,y) \text{ can be used as an error signal!} \]
How to follow the desired trajectory? We need the normal and tangent vectors.
We need the Jacobian of the path

Circle → \( f(x,y) := (x-x_c)^2 + (y-y_c)^2 - r^2 = 0 \)
Straight line → \( f(x,y) := ax + by + c = 0 \)

The Jacobian tells us in which direction the level set grows! (normal to the path)

Jacobian of the circle
\( \text{Jac} = 2[(x-x_c) (y-y_c)] \) is the normal vector to the circle

Jacobian of the straight line
\( \text{Jac} = [a \ b] \) is the normal vector to the line
We need the tangent to the path

It is just the 90 degrees rotation of the Jacobian

Tangent = Rot(90) Jac
The direction to follow is the combination of the normal and tangent

Control action or direction to follow

\( e := f(x,y) \) (error signal)

Direction to follow = tangent - e*normal
Example with an ellipse